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Nano $\delta\beta$-Open Sets in Nano Tritopological Spaces

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ARTICLE INFO. **ABSTRACT**

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This paper mainly concerns with generalizing nano near open sets in nano tritopological spaces by introducing new type of sets called nano $\tau_{R_{123}}\delta\beta$ open sets. These sets are stronger than any type of the other nano near open sets. The main properties and the relationships among these sets are discussed. In addition, the various forms of nano $\tau_{R_{123}}\delta\beta$-open sets corresponding to different cases of approximations are investigated. Moreover, the notion of nano $\tau_{R_{123}}\delta\beta$-continuous function is presented and compared to the other types of nano continuous functions.

Keywords: nano $\delta\beta$ open, nano $\delta\beta$ interior, nano $\delta\beta$ closure, nano $\delta\beta$ continuous, upper and lower approximation

1. Introduction

The study of tritopological space was first initiated by (Martin Kovar 2000). Separation axioms in tritopological spaces and the definition of 123 open set in tritopological spaces were studied by (Palaniammal 2011) and (Hameed 2011). (Tapi et. al 2016) proposed semi open and pre-open set in tritopological space. (Priyadharsini and Parvathi 2017) inrtoduced Tri-b-continuous function in tritopological spaces and studied their properties. (Thivagar and Richard 2013) presented the notion of Nano topology, which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it. They characterized the subset generated by lower approximations by certain objects that definitely form part of an interest subset, whereas the upper approximation is characterized by uncertain objects that possibly form part of an interest subset. The elements of a nano topological space are called the nano open sets. They also defined nano closed sets, nano interior and nano closure. Moreover, they introduced the weak forms of nano open sets namely nano α-open sets, nano semi-open sets, nano pre-open sets and nano regular open sets. (Revathy and Ilango 2015) generalized the previous work (Thivagar and Richard 2013) by introducing the concept of nano β-open sets. In 2016, Nasef et al. studied some properties for near nano open (closed) sets. (Thivagar and Richard 2013) introduced the notion of nano continuity in nano topological spaces. Thereafter, (Mary and Arockiarani 2015 and 2014) presented nano α-continuous, nano semi-continuous, nano pre-continuous, and nano b -continuous. (Nasef et al. 2016) investigated nano β – continuity.

The main aim of this paper is generalizing the previous near nano open sets and nano continuous 3functions in tritopological spaces. The remainder of this paper is arranged as follows. The fundamental concepts of nano tritopological spaces are introduced in Section 2. In Section 3 a new near nano open sets in tritopological spaces namely, nano $\tau_{R_{123}}\delta\beta$-open sets were introduced, and their properties were discussed. In addition, these sets are compared to the previous one and shown to be more general. Moreover, the concepts of nano $\tau_{R_{123}}\delta\beta$-closure and nano $\tau_{R_{123}}\delta\beta$-interior are obtained. In Section 4, various forms of nano $\tau_{R_{123}}\delta\beta$-open sets corresponding to different cases of approximations are investigated. A new class of functions on nano tritopological spaces called nano $\tau_{R_{123}}\delta\beta$ -continuous functions are presented in section 5. Furthermore, nano $\tau_{R_{123}}\delta\beta$-continuous functions and their characterizations are studied in terms of nano closed sets, nano closure, nano interior, nano $\tau_{R_{123}}\delta\beta$-closed sets, nano $\tau_{R_{123}}\delta\beta$-closure and nano $\tau_{R_{123}}\delta\beta$-interior. Relationships between the current functions and the previous one are analyzed. Finally, this paper concludes in Section 6.

2. Preliminaries

This section introduces the basic concepts of nano tritopological spaces, nano near tri-open sets, nano near tri-continuous and their properties.

Definition 2.1 (Pawlak 1982) Let *U* be a non-empty finite set of objects called the universe and *R* be an equivalence relation on *U*. the pair (*U*, *R*) is said to be the approximation space. Let $X \subseteq U$:

1) The lower approximation of *X* with respect to *R* is denoted by $L_R(X) = \cup_{x\in U} \{R(X): R(X) \subseteq X\}$. Where $R(X)$ denotes the equivalence class determined by *x*.
2) The upper approximation of *X* with respect to *R* is denoted by $H_R(X) = \cup_{x\in U} \{R(X): R(X) \cap X \neq \emptyset\}$.
3) The boundary region of *X* with respect to *R* is denoted by $B_R(X) = H_R(X) - L_R(X)$.

Definition 2.2 (Thivagar and Richard 2013) Let *U* be the universe, *R* be an equivalence relation on *U* and $X \subseteq U$. The nano topology on *U* with respect to *X* is defined by $\tau_R(X) = \{U, \emptyset, H_R(X), L_R(X), B_R(X)\}$ and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano open sets and the complement of nano open set is called nano closed set. The family of all nano $\tau_R$open sets is denoted by nano $\tau_R O(U, X)$ and the family of all nano $\tau_R$closed sets is denoted by nano $\tau_R C(U, X)$.

Definition 2.3 (Chandrasekar et. al 2017) Let *U* be the universe set, R_1, R_2 and R_3 are equivalence relations on U and $X \subseteq U$. If $(U, \tau_{R_1}(X))$, $(U, \tau_{R_2}(X))$ and $(U, \tau_{R_3}(X))$ are three nano topological spaces with respect to X. Define the nano topology $N\tau^*_{R_{123}}$called nano tri star topology induced by two nano bitopology, as follows:

$$\tau^*_{R_{123}}(X) = \left[\tau_{R_1}(X) \cup \tau_{R_3}(X)\right] \cap \left[\tau_{R_2}(X) \cup \tau_{R_3}(X)\right]$$

Then the space $(U, \tau_{R_1}(X), \tau_{R_2}(X), \tau_{R_3}(X))$ (briefly, $\left(U, \tau^*_{R_{123}}(X)\right)$) is called a nano tritopological space.

The three nano topological spaces $(U, \tau_{R_1}(X))$, $(U, \tau_{R_2}(X))$ and $(U, \tau_{R_3}(X))$ are independently satisfy the axioms of nano topological space.

Definition 2.4 (Chandrasekar et. al 2017) The elements of $N\tau^*_{R_{123}}$ are called $N\tau^*_{R_{123}}$ open sets and the complement of $N\tau^*_{R_{123}}$ open set is called $N\tau^*_{R_{123}}$closed set.

Definition 2.5 (Tapi et. al 2016) A subset *A* of a nano tritopological space $\left(U, \tau_{R_{123}}(X)\right)$ is called,

1) Nano $\tau_{R_{123}}$regular open if $A =$ nano $\tau_{R_{123}}int($nano $\tau_{R_{123}}cl(A))$,
2) Nano $\tau_{R_{123}}$semi open if $A \subseteq$ nano $\tau_{R_{123}}cl($nano $\tau_{R_{123}}int(A))$,
3) Nano $\tau_{R_{123}}$preopen if $A \subseteq$ nano $\tau_{R_{123}}int($nano $\tau_{R_{123}}cl(A))$,
4) Nano $\tau_{R_{123}}\alpha$-open if $A \subseteq$ nano $\tau_{R_{123}}int$ (nano $\tau_{R_{123}}cl\big($nano $\tau_{R_{123}}int\ (A)\big))$,
5) Nano $\tau_{R_{123}}b$-open if $A \subseteq$ [nano $\tau_{R_{123}}int$ (nano $\tau_{R_{123}}cl(A)] \cup$ [nano $\tau_{R_{123}}cl\big($nano $\tau_{R_{123}}int\ A)\big)]$,
6) Nano $\tau_{R_{123}}\beta$-open if $A \subseteq$ nano $\tau_{R_{123}}cl$ (nano $\tau_{R_{123}}int\big($nano $\tau_{R_{123}}cl(A)\big)$.

The family of all nano $\tau_{R_{123}}$regular open (respectively, nano $\tau_{R_{123}}$semi open, nano $\tau_{R_{123}}$preopen, nano $\tau_{R_{123}}\alpha$-open, nano $\tau_{R_{123}}b$-open and nano $\tau_{R_{123}}\beta$-open) sets in a nano tritopological space $\left(U, \tau_{R_{123}}(X)\right)$ is denoted by nano $\tau_{R_{123}}RO(U, X)$ (respectively, nano $\tau_{R_{123}}SO(U, X)$, nano $\tau_{R_{123}}PO(U, X)$, nano $\tau_{R_{123}}\alpha O(U, X)$, nano $\tau_{R_{123}}bO(U, X)$ and nano $\tau_{R_{123}}\beta O(U, X)$).

The complement of nano $\tau_{R_{123}}$regular open (respectively, nano $\tau_{R_{123}}$semi open, nano $\tau_{R_{123}}b$-open and nano $\tau_{R_{123}}\beta$-open) set is defined to be nano $\tau_{R_{123}}$regular closed (respectively, nano $\tau_{R_{123}}$semi closed, nano $\tau_{R_{123}}b$- closed and nano $\tau_{R_{123}}\beta$- closed) set.

3. Nano $\boldsymbol{\tau_{R_{123}}\delta\beta}$- open sets

The goal of this section is to study the concept of nano $\tau_{R_{123}}\delta\beta$-open sets and their properties in tritopological spaces. It is clarified that this concept is an extension of the previous concepts of nano near $\tau_{R_{123}}$open sets. Several properties of this concept are studied.

Definition 3.1 Let U be the universe set, R_1, R_2 and R_3 are equivalence relations on U and $X \subseteq U$. If $\left(U, \tau_{R_1}(X)\right)$, $\left(U, \tau_{R_2}(X)\right)$ and $\left(U, \tau_{R_3}(X)\right)$ are three nano topological spaces with respect to *X*. Define the nano tri topology $N\tau_{R_{123}}(X)$ as follows,

$N\tau_{R_{123}}(X) = \tau_{R_1}(X) \cup \tau_{R_2}(X) \cup \tau_{R_3}(X)$. Then $(U, \tau_{R_1}(X), \tau_{R_2}(X), \tau_{R_3}(X))$ (briefly, $\left(U, \tau_{R_{123}}(X)\right)$) is called a nano tritopological space with respect to X.

The three nano topological spaces $(U, \tau_{R_1}(X))$, $(U, \tau_{R_2}(X))$ and $(U, \tau_{R_3}(X))$ are independently satisfy the axioms of nano topological space.

Definition 3.2 The elements of $N\tau_{R_{123}}(X)$ are called $N\tau_{R_{123}}$ open sets and the complement of $N\tau_{R_{123}}$ open set is called $N\tau_{R_{123}}$ closed set.

Definition 3.3 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to *X*, $A \subseteq U$ then,

1) The nano $\tau_{R_{123}}$ interior of *A* is defined as the union of all nano $\tau_{R_{123}}$ open subsets contained in *A* and it is denoted by $N\tau_{R_{123}} - int(A) = \cup\{G: G \subseteq A$ and G is $N\tau_{R_{123}}$open subset$\}$. $N\tau_{R_{123}}int(A)$ is the largest $N\tau_{R_{123}}$ open subsets of A.
2) The nano $\tau_{R_{123}}$ closure of *A* is defined as the intersection of all $N\tau_{R_{123}}$ closed sets containing *A* and it is denoted by $N\tau_{R_{123}} - cl(A) = \cap\{F: F \supseteq A$ and F is $N\tau_{R_{123}}$closed subset$\}$.

Definition 3.4 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to *X*, and $A \subseteq U$, the nano $\tau_{R_{123}}\delta$-interior of *A* which denoted by $N\tau_{R_{123}} - int_\delta(A)$ is the union of all nano $\tau_{R_{123}}$regular open sets of *U* contained in *A*. A $N\tau_{R_{123}}$subset *A* is called $N\tau_{R_{123}} - \delta$-open if $A = N\tau_{R_{123}} - int_\delta(A)$.

Definition 3.5 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to *X* and $A \subseteq U$ then the nano $\tau_{R_{123}}\delta$-closure of the set *A* which denoted by $N\tau_{R_{123}} - cl_\delta(A)$ is defined by $N\tau_{R_{123}} - cl_\delta(A) = \{x \in U: A \cap N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl(B)\right) \neq \emptyset,$ $B \in N\tau_{R_{123}} - O(X)$ and $x \in B\}$. A $N\tau_{R_{123}}$ subset *A* is called $N\tau_{R_{123}} - \delta$ -closed if $A = N\tau_{R_{123}} - cl_\delta(A)$.

Note that:

$N\tau_{R_{123}} - int_\delta(A) = \left(N\tau_{R_{123}} - cl_\delta(A^c)\right)^c$.

Definition 3.6 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to *X*, and $A \subseteq U$. The set *A* is said to be:

1) $N\tau_{R_{123}} - \delta$-regular open if $A = N\tau_{R_{123}} - int(N\tau_{R_{123}} - cl_\delta(A))$,
2) $N\tau_{R_{123}} - \delta$-semi open if $A \subseteq N\tau_{R_{123}} - cl(N\tau_{R_{123}} - int_\delta(A))$,
3) $N\tau_{R_{123}} - \delta$-preopen if $A \subseteq N\tau_{R_{123}} - int(N\tau_{R_{123}} - cl_\delta(A))$,
4) $N\tau_{R_{123}} - \delta\alpha$-open if $A \subseteq N\tau_{R_{123}} - int(N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int_\delta(A)\right))$.

The complement of $N\tau_{R_{123}} - \delta$-regular open (respectively, $N\tau_{R_{123}} - \delta$-semi open, $N\tau_{R_{123}} - \delta$-preopen and $N\tau_{R_{123}} - \delta\alpha$-open) set is defined to be $N\tau_{R_{123}} - \delta$-regular closed (respectively, $N\tau_{R_{123}}$- δ semi closed, $N\tau_{R_{123}} - \delta$-pre-closed and $N\tau_{R_{123}} - \delta\alpha$-closed) set.

Definition 3.7 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to *X* and $A \subseteq U$, then *A* is called a nano $\tau_{R_{123}}\delta\beta$- open set which denoted by $N\tau_{R_{123}} - \delta\beta$-open set if $A \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right)$. The complement of a $N\tau_{R_{123}} - \delta\beta$-open set is called $N\tau_{R_{123}}$- $\delta\beta$-closed set. The family of all $N\tau_{R_{123}}$- $\delta\beta$- open (resp. $N\tau_{R_{123}}$ - $\delta\beta$ -closed) sets of $\left(U, \tau_{R_{123}}(X)\right)$ is denoted by $N\tau_{R_{123}} - \delta\beta O(U, X)$ (resp. $N\tau_{R_{123}} - \delta\beta C(U, X)$).

Definition 3.8 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to *X* and $A \subseteq U$. The nano $\tau_{R_{123}}\delta\beta$-closure of a set *A*, denoted by $N\tau_{R_{123}} - cl_{\delta\beta}(A)$, is the intersection of $N\tau_{R_{123}} - \delta\beta$-closed sets containing *A*. The nano $\tau_{R_{123}}\delta\beta$-interior of a set *A*, denoted by $N\tau_{R_{123}} - int_{\delta\beta}(A)$, is the union of $N\tau_{R_{123}} - \delta\beta$-open sets contained in *A*.

Proposition 3.1 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to *X* and $A \subseteq U$, then $N\tau_{R_{123}} - cl(A) \subseteq N\tau_{R_{123}} - cl_\delta(A)$.

Proof. Let $x \in N\tau_{R_{123}} - cl(A)$. Then $G \cap A \neq \emptyset, \forall G, x \in G, G \in \tau_{R_{123}}(X)$. Therefore, $A \cap G = A \cap N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - int(G)\right) \subseteq A \cap N\tau_{R_{123}} - int(N\tau_{R_{123}} - cl(G))$. Thus $A \cap N\tau_{R_{123}} - int(N\tau_{R_{123}} - cl(G)) \neq \emptyset$. Hence $N\tau_{R_{123}} - cl(A) \subseteq N\tau_{R_{123}} - cl_\delta(A)$.

Theorem 3.1 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to *X* and $A, B \subseteq U$. Then the following properties hold:

1) $A \subseteq N\tau_{R_{123}} - cl_\delta(A)$.

2) If $A \subseteq B$, then $N\tau_{R_{123}} - cl_\delta(A) \subseteq N\tau_{R_{123}} - cl_\delta(B)$.

3) $N\tau_{R_{123}} - cl_\delta(A \cap B) \subseteq \left(N\tau_{R_{123}} - cl_\delta(A)\right) \cap \left(N\tau_{R_{123}} - cl_\delta(B)\right)$.

4) $N\tau_{R_{123}} - cl_\delta(A \cup B) = \left(N\tau_{R_{123}} - cl_\delta(A)\right) \cup \left(N\tau_{R_{123}} - cl_\delta(B)\right)$.

Proof. Obvious.

Proposition 3.2 Every $N\tau_{R_{123}} - \beta$-open set is nano $\tau_{RN\tau_{R_{123}}-_{123}}\delta\beta$-open set.

Proof. By using the properties of nano interior, nano closure and **Proposition 3.1**, the proof is obvious.

Proposition 3.2 shows that $N\tau_{R_{123}} - \delta\beta$-open sets are generalization of $N\tau_{R_{123}} - \beta$-open sets. Consequently, it is stronger than all the previous nano near open sets.

The relationships between $N\tau_{R_{123}} - \delta\beta$-open sets and the other nano near open sets is shown in the following diagram,

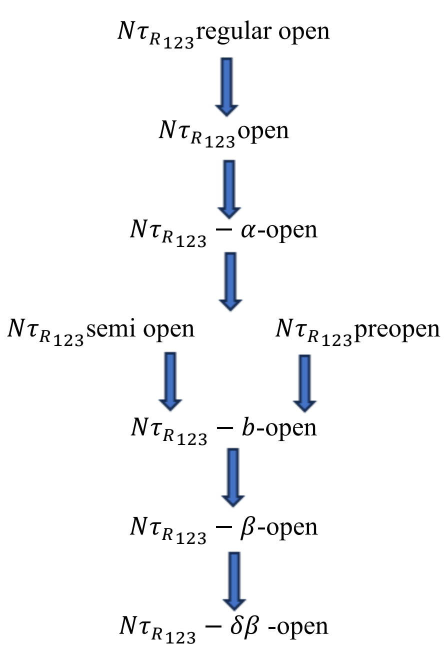

Figure 1: The relationships between $N\tau_{R_{123}} - \delta\beta$ open sets and the other nano near open sets.

Remark 3.1 The converse of **Proposition 3.2** is not necessarily true as shown in the following example.

Example 3.1 Let $U = \{a, b, c\}$, $X = \{a, b\}$ with

$U/R_1 = \{\{a\}, \{b\}, \{c\}\}$ then $\tau_{R_1}(X) = \{U, \emptyset, \{a, b\}\}$,

$U/R_2 = \{\{a, b\}, \{c\}\}$ then $\tau_{R_2}(X) = \{U, \emptyset, \{a, b\}\}$.

$U/R_3 = \{\{a, c\}, \{b\}\}$ then $\tau_{R_3}(X) = \{U, \emptyset, \{b\}, \{a, c\}\}$. Then, $N\tau_{R_{123}}(X) = \tau_{R_1}(X) \cup \tau_{R_2}(X) \cup \tau_{R_3}(X) = \{U, \emptyset, \{b\}, \{a, b\}, \{a, c\}\}$.

Now we have, $\{c\}$ is a $N\tau_{R_{123}} - \delta\beta$ open set, but it is neither a $N\tau_{R_{123}} - \beta$ open set, $N\tau_{R_{123}} - b$ open set nor $N\tau_{R_{123}}$ preopen set. $\{b, c\}$ is a $N\tau_{R_{123}} - \delta\beta$ open set, but it is not $N\tau_{R_{123}} - \alpha$ open set. $\{a\}$ is $N\tau_{R_{123}} - \delta\beta$ open set, but it is neither a $N\tau_{R_{123}}$ regular open set nor $N\tau_{R_{123}}$ semi open set.

Proposition 3.3 The union of $N\tau_{R_{123}} - \delta\beta$-open sets is also $N\tau_{R_{123}} - \delta\beta$-open sets.

Proof. Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to X and $A, B \subseteq U$ are two $N\tau_{R_{123}} - \delta\beta$-open sets. Then we have $A \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right)$ and $B \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(B)\right)\right)$. Thus $A \cup B \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) \cup N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(B)\right)\right) = N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right) \cup N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(B)\right)\right) \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A) \cup N\tau_{R_{123}} - cl_\delta(B)\right)\right) = N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A \cup B)\right)\right)$.

By **Theorem 3.1** (4).

Therefore, $A \cup B \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A \cup B)\right)\right)$. Hence, $A \cup B$ is also a $N\tau_{R_{123}} - \delta\beta$-open set.

Remark 3.2 The family of all $N\tau_{R_{123}} - \delta\beta$ -open sets in U does not form a topology, as the intersection of two $N\tau_{R_{123}} - \delta\beta$-opensets need not be $N\tau_{R_{123}} - \delta\beta$-open set as shown in the following example.

Example 3.2 Let $U = \{a,b,c,d\}$, $X = \{a,c\}$ with $U/R_1 = \{\{a,b\},\{c\},\{d\}\}$ then $\tau_{R_1}(X) = \{U,\emptyset,\{c\},\{a,b\},\{a,b,c\}\}$. $U/R_2 = \{\{a,b,c\},\{d\}\}$ then $\tau_{R_2}(X) = \{U,\emptyset,\{a,b,c\}\}$. $U/R_3 = \{\{a,b\},\{c,d\}\}$ then $\tau_{R_3}(X) = \{U,\emptyset\}$. Hence, $N\tau_{R_{123}}(X) = \tau_{R_1}(X) \cup \tau_{R_2}(X) \cup \tau_{R_3}(X) = \{U,\emptyset,\{c\},\{a,b\},\{a,b,c\}\}$.

The family of all $N\tau_{R_{123}} - \delta\beta$ open sets in U is $P(U) - \{d\}$. Clearly this family does not form a topology.

If $A = \{b,d\}, B = \{c,d\}$, then A, B are two $N\tau_{R_{123}} - \delta\beta$ open sets, but $A \cap B = \{d\}$ is not a $N\tau_{R_{123}} - \delta\beta$ open set.

Corollary 3.1 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to *X*. Then $N\tau_{R_{123}} - SO(U,X) \cup N\tau_{R_{123}} - PO(U,X) \subseteq N\tau_{R_{123}} - \delta\beta O(U,X)$.

Remark 3.3 The equality in **Corollary 3.1** does not hold in general as shown in the following example.

Example 3.3 From **Example 3.1**, $A = \{c\}$ is a $N\tau_{R_{123}} - \delta\beta$-open set but not in the union of $N\tau_{R_{123}} - SO(U,X)$ and $N\tau_{R_{123}} - PO(U,X)$.

Remark 3.4 The arbitrary intersection of $N\tau_{R_{123}} - \delta\beta$-closed sets is $N\tau_{R_{123}} - \delta\beta$-closed, but the union of two $N\tau_{R_{123}} - \delta\beta$-closed sets may not be a $N\tau_{R_{123}} - \delta\beta$-closed set as shown in the following example.

Example 3.4 Let $U = \{a,b,c,d\}$, $X = \{a,d\}$ with $U/R_1 = \{\{a\},\{b,c\},\{d\}\}$ then $\tau_{R_1}(X) = \{U,\emptyset,\{a,d\}\}$, $U/R_2 = \{\{a\},\{b\},\{c,d\}\}$ then $\tau_{R_2}(X) = \{U,\emptyset,\{a\},\{c,d\},\{a,c,d\}\}$. $U/R_3 = \{\{b\},\{a,c,d\}\}$ then $\tau_{R_3}(X) = \{U,\emptyset,\{a,c,d\}\}$.

Hence, $N\tau_{R_{123}}(X) = \tau_{R_1}(X) \cup \tau_{R_2}(X) \cup \tau_{R_3}(X) = \{U,\emptyset,\{a\},\{a,d\},\{c,d\},\{a,c,d\}\}$.

If $A = \{a\}, B = \{c,d\}$, then A, B are two $N\tau_{R_{123}} - \delta\beta$ closed sets, but $A \cup B = \{a,c,d\}$ is not a $N\tau_{R_{123}} - \delta\beta$ closed set.

Proposition 3.4 The intersection of $N\tau_{R_{123}}$open and $N\tau_{R_{123}} - \delta\beta$-open is $N\tau_{R_{123}} - \delta\beta$-open.

Proof. Let A be $N\tau_{R_{123}}$open and B be $N\tau_{R_{123}} - \delta\beta$-open. Then,

$A \cap B \subseteq A \cap N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(B)\right)\right) \subseteq N\tau_{R_{123}} - cl\left(A \cap N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(B)\right)\right) \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A \cap B)\right)\right)$.

Therefore, $A \cap B$ is $N\tau_{R_{123}} - \delta\beta$-open set.

Corollary 3.2 The union of $N\tau_{R_{123}}$closed and $N\tau_{R_{123}} - \delta\beta$-closed is $N\tau_{R_{123}} - \delta\beta$-closed.

Proposition 3.5 If *A* and *B* are $N\tau_{R_{123}} - \delta\beta$-open subsets of *U* such that $A \subseteq B \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int(A)\right)$, then *B* is $N\tau_{R_{123}} - \delta\beta$-open in *U*.

Proof. Since $N\tau_{R_{123}} - int(A) \subseteq N\tau_{R_{123}} - cl(A)$ then, $N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - int(A)\right) \subseteq N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl(A)\right)$. Which implies that $N\tau_{R_{123}} - int(A) \subseteq N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl(A)\right)$.

Then, $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int(A)\right) \subseteq N\tau_{R_{123}} - cl(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl(A)\right)) \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right)$.

By **Proposition 3.1**.

Since, $B \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int(A)\right) \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right)$.

By **Theorem 3.1** (1).

$\subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(B)\right)\right)$

By **Theorem 3.1** (2).

Hence, $B \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(B)\right)\right)$. Therefore, *B* is $N\tau_{R_{123}} - \delta\beta$-open in *U*.

Remark 3.5 Each $N\tau_{R_{123}} - \delta$-regular open set (respectively, $N\tau_{R_{123}} - \delta$-semi open set, $N\tau_{R_{123}} - \delta$-preopen set and $N\tau_{R_{123}} - \delta\alpha$-open set) is a $N\tau_{R_{123}} - \delta\beta$-open but the converse does not hold in general as shown in the following example.

Example 3.5 From **Example 3.1.2**:

$\{a,c,d\}$ is a $N\tau_{R_{123}} - \delta\beta$ open set, but it is not $N\tau_{R_{123}} - \delta$ open set.

$\{b\}$ is a $N\tau_{R_{123}} - \delta\beta$ open set, but it is not $N\tau_{R_{123}} - \delta$ regular set.

$\{a, d\}$ is a $N\tau_{R_{123}} - \delta\beta$ open set, but it is not $N\tau_{R_{123}} - \delta$ preopen set.

$\{a\}$ is a nano $N\tau_{R_{123}} - \delta\beta$ open set, but it is not $N\tau_{R_{123}} - \delta$ semi open set.

$\{b, c\}$is a nano $N\tau_{R_{123}} - \delta\beta$ open set, but it is not $N\tau_{R_{123}} - \delta\alpha$ open set.

Proposition 3.6 Every $N\tau_{R_{123}} - \delta\beta$-open set which is,

1) $N\tau_{R_{123}}$-δ-semi closed is $N\tau_{R_{123}}$-semi open set,
2) $N\tau_{R_{123}} - \delta\alpha$- closed is $N\tau_{R_{123}}$ −closed set,
3) $N\tau_{R_{123}} - \delta\alpha$- closed is $N\tau_{R_{123}}$ −regular closed set.

Proof.

1) Let $A \subseteq U$ be $N\tau_{R_{123}} - \delta\beta$-open and $N\tau_{R_{123}} - \delta$-semi closed set. Then, $A \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right)$ and $N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right) \subseteq A$.
Hence, $N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right) \subseteq A \subseteq N\tau_{R_{123}} - cl(N\tau_{R_{123}} - int(N\tau_{R_{123}} - cl_\delta(A))$.
Since,$N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right) \subseteq N\tau_{R_{123}} - int(A)$, which implies that $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int(A)\right)$.

Then, $A \subseteq N\tau_{R_{123}} - cl(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)) \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int(A)\right)$. Hence, $A \subseteq N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int(A)\right)$. This implies that A is $N\tau_{R_{123}}$semi open set.

2) Let $A \subseteq U$ be $N\tau_{R_{123}} - \delta\beta$-open and $N\tau_{R_{123}} - \delta\alpha$ closed set. Then, $A \subseteq N\tau_{R_{123}} - cl(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right))$ and $N\tau_{R_{123}} - cl(N\tau_{R_{123}} - int(N\tau_{R_{123}} - cl_\delta(A)) \subseteq A$. Hence, $A = N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right)$. This implies that A is $N\tau_{R_{123}} -$ closed set.

3) Obvious.

Corollary 3.3 Every $N\tau_{R_{123}} - \delta\beta$-closed set which is,

1) $N\tau_{R_{123}} - \delta$-semi open is $N\tau_{R_{123}}$semi closed set,
2) $N\tau_{R_{123}} - \delta\alpha$- open is $N\tau_{R_{123}}$open set,
3) $N\tau_{R_{123}} - \delta\alpha$- open is $N\tau_{R_{123}}$regular open set.

Theorem 3.2 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to X and $A, B \subseteq U$. Then the following properties holds:

1) $N\tau_{R_{123}} - \delta\beta$-closure is a $N\tau_{R_{123}} - \delta\beta$-closed set and $N\tau_{R_{123}} - \delta\beta$-interior is a $N\tau_{R_{123}} - \delta\beta$-open set.
2) $A \subseteq N\tau_{R_{123}} - cl_{\delta\beta}(A)$.
3) $A = N\tau_{R_{123}} - cl_{\delta\beta}(A)$ iff A is a $N\tau_{R_{123}} - \delta\beta$-closed set.
4) $N\tau_{R_{123}} - int_{\delta\beta}(A) \subseteq A$.
5) $A = N\tau_{R_{123}} - int_{\delta\beta}(A)$ iff A is a $N\tau_{R_{123}} - \delta\beta$-open set.
6) If $A \subseteq B$, then $N\tau_{R_{123}} - cl_{\delta\beta}(A) \subseteq N\tau_{R_{123}} - cl_{\delta\beta}(B)$ and $N\tau_{R_{123}} - int_{\delta\beta}(A) \subseteq N\tau_{R_{123}} - int_{\delta\beta}(B)$.
7) $\left(N\tau_{R_{123}} - int_{\delta\beta}(A)\right) \cup \left(N\tau_{R_{123}} - int_{\delta\beta}(B)\right) \subseteq N\tau_{R_{123}} - int_{\delta\beta}(A \cup B)$.
8) $N\tau_{R_{123}} - cl_{\delta\beta}(A \cap B) \subseteq \left(N\tau_{R_{123}} - cl_{\delta\beta}(A)\right) \cap \left(N\tau_{R_{123}} - cl_{\delta\beta}(B)\right)$.

Proof. Obvious

Remark 3.6 The inclusion in **Theorem 3.2** parts 2, 4, 7 and 8 cannot be replaced by equality relation. Also, the converse of part 6 is not necessarily true. This can be shown as in the following example.

Example 3.6 From **Example 3.4**:

For part 2). If $A = \{a, c, d\}$, $N\tau_{R_{123}} - cl_{\delta\beta}(A) = U$, then $N\tau_{R_{123}} - cl_{\delta\beta}(A) \neq A$.

For part 4). If $A = \{b\}$, $N\tau_{R_{123}} - int_{\delta\beta}(A) = \emptyset$, then $A \nsubseteq N\tau_{R_{123}} - int_{\delta\beta}(A)$.

For part 7). If $A = \{b\}, B = \{c, d\}$, $A \cup B = \{b, c, d\}$. Then $N\tau_{R_{123}} - int_{\delta\beta}(A) = \emptyset$, $N\tau_{R_{123}} - int_{\delta\beta}(B) = \{c, d\}, N\tau_{R_{123}} - int_{\delta\beta}(A \cup B) = \{b, c, d\}$.

Hence, $N\tau_{R_{123}} - \left(int_{\delta\beta}(A \cup B)\right) = \{b, c, d\} \nsubseteq \{c, d\} = N\tau_{R_{123}} - int_{\delta\beta}(A) \cup N\tau_{R_{123}} - int_{\delta\beta}(B)$.

For part 8). If $A = \{b\}, B = \{a, c, d\}$. Then, $A \cap B = \emptyset, N\tau_{R_{123}} - cl_{\delta\beta}(A) = \{b\}$, $N\tau_{R_{123}} - cl_{\delta\beta}(B) = U, N\tau_{R_{123}} cl_{\delta\beta} - (A \cap B) = \emptyset$. Hence $\{b\} = N\tau_{R_{123}} - cl_{\delta\beta}(A) \cap N\tau_{R_{123}} - cl_{\delta\beta}(B) \nsubseteq N\tau_{R_{123}} - cl_{\delta\beta}(A \cap B) = \emptyset$.

For part 6). If $A = \{a, b\}, B = \{a, c, d\}$. Then, $N\tau_{R_{123}} - cl_{\delta\beta}(A) = \{a, b\}, N\tau_{R_{123}} - cl_{\delta\beta}(B) = U$.

Therefore, $N\tau_{R_{123}} - cl_{\delta\beta}(A) \subseteq N\tau_{R_{123}} - cl_{\delta\beta}(B)$ but $A \nsubseteq B$. If $A = \{b\}, B = \{c, d\}$. Then, $N\tau_{R_{123}} - int_{\delta\beta}(A) = \emptyset, N\tau_{R_{123}} - int_{\delta\beta}(B) = \{c, d\}$. Therefore, $N\tau_{R_{123}} - int_{\delta\beta}(A) \subseteq N\tau_{R_{123}} - int_{\delta\beta}(B)$ but $A \nsubseteq B$.

4. Nano $\tau_{R_{123}}\delta\beta$- open sets and upper & lower approximations

This section investigates the various forms of $N\tau_{R_{123}} - \delta\beta$-open sets corresponding to different cases of approximations.

Proposition 4.1 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to X. If $H_{R_{123}}(X) = U$, then $N\tau_{R_{123}} - \delta\beta O(U, X)$ is $P(U)$.

Proof. Let $H_{R_{123}}(X) = U$.

a) If $L_{R_{123}}(X) = \emptyset$, then $B_{R_{123}}(X) = U$ and $\tau_{R_{123}}(X) = \{U, \emptyset\}$. Hence, $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = U, \forall A \subseteq U$.

Thus, $A \subseteq N\tau_{R_{123}} - cl(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)), \forall A \subseteq U$.

Therefore A is $N\tau_{R_{123}} - \delta\beta$-open in U. Hence, $N\tau_{R_{123}} - \delta\beta O(U, X)$ is $P(U)$.

b) If $L_{R_{123}}(X) \neq \emptyset$, then $B_{R_{123}}(X) = U - L_{R_{123}}(X) = \left(L_{R_{123}}(X)\right)^c$.

Hence, $\tau_{R_{123}}(X) = \{U, \emptyset, L_{R_{123}}(X), \left(L_{R_{123}}(X)\right)^c\}$:

1) If $A \subseteq L_{R_{123}}(X)$ then $N\tau_{R_{123}} - cl_\delta(A) = L_{R_{123}}(X)$ and $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = L_{R_{123}}(X)$. Therefore A is $N\tau_{R_{123}} - \delta\beta$-open in U.
2) If $A \subseteq \left(L_{R_{123}}(X)\right)^c$, then $N\tau_{R_{123}} - cl_\delta(A) = \left(L_{R_{123}}(X)\right)^c$, and $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = \left(L_{R_{123}}(X)\right)^c$. Therefore A is $N\tau_{R_{123}} - \delta\beta$-open in U.
3) If $A \cap L_{R_{123}}(X) \neq \emptyset$ and $A \cap \left(L_{R_{123}}(X)\right)^c \neq \emptyset$, then $N\tau_{R_{123}} - cl_\delta(A) = U$, and $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = U$. Therefore A is $N\tau_{R_{123}} - \delta\beta$-open in U.

Proposition 4.2 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to *X*. If $H_{R_{123}}(X) \neq U$, and $\left(L_{R_{123}}(X) = \emptyset \; or \; L_{R_{123}}(X) = H_{R_{123}}(X)\right)$ then $N\tau_{R_{123}} - \delta\beta O(U, X)$ is $P(U)$.

Proof. Let $H_{R_{123}}(X) \neq U$, and $\left(L_{R_{123}}(X) = \emptyset \; or \; L_{R_{123}}(X) = H_{R_{123}}(X)\right)$. In both cases $\tau_{R_{123}}(A) = \{U, \emptyset, H_{R_{123}}(X)\}$.

Hence, $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = U, \forall A \subseteq U$. Thus A is $N\tau_{R_{123}} - \delta\beta$-open $\forall A \subseteq U$. Therefore $N\tau_{R_{123}} - \delta\beta O(U, X)$ is $P(U)$.

Proposition 4.3 Let $\left(U, \tau_{R_{123}}(X)\right)$ be a nano tritopological space with respect to X. If $H_{R_{123}}(X) \neq U$, and $L_{R_{123}}(X) \neq \emptyset$ then $U, \emptyset$ and any set which intersects $H_{R_{123}}(X)$ are $N\tau_{R_{123}} - \delta\beta$-open sets in U.

Proof. Let $H_{R_{123}}(X) \neq U$, and $L_{R_{123}}(X) \neq \emptyset$, then $\tau_{R_{123}}(A) = \{U, \emptyset, H_{R_{123}}(X), L_{R_{123}}(X), B_{R_{123}}(X)\}$:

a) If $A \subseteq H_{R_{123}}(X)$:

1) If $A \subseteq L_{R_{123}}(X)$, then $N\tau_{R_{123}} - cl_\delta(A) = L_{R_{123}}(X)$, $N\tau_{R_{123}} - \tau_{R_{123}} cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = \left(B_{R_{123}}(X)\right)^c$. Hence, $A \subseteq L_{R_{123}}(X) \subseteq \left(B_{R_{123}}(X)\right)^c = N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right)$. Therefore A is $N\tau_{R_{123}} - \delta\beta$-open in U.

2) If $A \subseteq B_{R_{123}}(X)$, then $N\tau_{R_{123}} - cl_\delta(A) = B_{R_{123}}(X)$ and $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = \left(L_{R_{123}}(X)\right)^c$. Hence, $A \subseteq B_{R_{123}}(X) \subseteq \left(L_{R_{123}}(X)\right)^c = N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right)$. Therefore A is $N\tau_{R_{123}} - \delta\beta$-open in U.
3) If $A \cap L_{R_{123}}(X) \neq \emptyset$ and $A \cap B_{R_{123}}(X) \neq \emptyset$, then $N\tau_{R_{123}} - cl_\delta(A) = U$, $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = U$. Therefore A is $N\tau_{R_{123}} - \delta\beta$-open in U.

b) If $A \subseteq \left(H_{R_{123}}(X)\right)^c$, then $N\tau_{R_{123}} - cl_\delta(A) = \left(H_{R_{123}}(X)\right)^c$. Hence, $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = \emptyset$. Therefore A is not $N\tau_{R_{123}} - \delta\beta$-open.

c) If $A \cap H_{R_{123}}(X) \neq \emptyset$ and $A \cap \left(H_{R_{123}}(X)\right)^c \neq \emptyset$:

1) If $A \cap L_{R_{123}}(X) \neq \emptyset$ and $A \cap \left(H_{R_{123}}(X)\right)^c \neq \emptyset$, then $N\tau_{R_{123}} - cl_\delta(A) = \left(B_{R_{123}}(X)\right)^c$, $N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right) = L_{R_{123}}(X)$, $N\tau_{R_{123}} - cl(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)) = \left(B_{R_{123}}(X)\right)^c = \left(H_{R_{123}}(X)\right)^c \cup L_{R_{123}}(X) \supseteq A$. Therefore A is $N\tau_{R_{123}} - \delta\beta$-open in U.
2) If $A \cap B_{R_{123}}(X) \neq \emptyset$ and $A \cap \left(H_{R_{123}}(X)\right)^c \neq \emptyset$, then $N\tau_{R_{123}} - cl_\delta(A) = \left(L_{R_{123}}(X)\right)^c$, $N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right) = B_{R_{123}}(X)$,$N\tau_{R_{123}} - cl(N\tau_{R_{123}} - int\ \left(N\tau_{R_{123}} - cl_\delta(A)\right)) = \left(H_{R_{123}}(X)\right)^c \cup B_{R_{123}}(X) \supseteq A$. Therefore A is nano $\tau_R N\tau_{R_{123}} -_{123} \delta\beta$-open in U.
3) If $A \cap L_{R_{123}}(X) \neq \emptyset$, $A \cap B_{R_{123}}(X) \neq \emptyset$ and $A \cap \left(H_{R_{123}}(X)\right)^c \neq \emptyset$, then $N\tau_{R_{123}} - cl_\delta(A) = U$ and $N\tau_{R_{123}} - cl\left(N\tau_{R_{123}} - int\left(N\tau_{R_{123}} - cl_\delta(A)\right)\right) = U$. Therefore A is $N\tau_{R_{123}} - \delta\beta$-open in U.

5. Nano $\tau_{R_{123}}\delta\beta$- continuity

In this section, the main properties, and characterizations of $N\tau_{R_{123}} - \delta\beta$-continues functions are presented. It is a generalization of the other types of $N\tau_{R_{123}}$near continues functions. Many important results are also obtained in this section.

Definition 5.1 Let U be a universe set, R_1, R_2 and R_3 be an equivalence relations on U, $X \subseteq U$ and V be a universe set, R_1', R_2' and R_3' be an equivalence relations on V, $Y \subseteq V$ and $\left(U, \tau_{R_{123}}(X)\right)$ and $\left(V, \sigma_{R'_{123}}(Y)\right)$ be a nano tritopological spaces. A function $f: \left(U, \tau_{R_{123}}(X)\right) \to \left(V, \sigma_{R'_{123}}(Y)\right)$ is said to be:

1) Nano $\tau_{R_{123}}$semi-continuous if $f^{-1}(H)$ is a nano $\tau_{R_{123}}$semi- open set in U for every nano $\sigma_{R'_{123}}$open set H in V.
2) Nano $\tau_{R_{123}}$pre-continuous if $f^{-1}(H)$ is a nano $\tau_{R_{123}}$preopen set in U for every nano $\sigma_{R'_{123}}$open set H in V.
3) Nano $\tau_{R_{123}}\alpha$-continuous if $f^{-1}(H)$ is a nano $\tau_{R_{123}}\alpha$-open set in U for every nano $\sigma_{R'_{123}}$open set H in V.
4) Nano $\tau_{R_{123}}b$-continuous if $f^{-1}(H)$ is a nano $\tau_{R_{123}}\gamma$-open set in U for every nano $\sigma_{R'_{123}}$open set H in V.
5) Nano $\tau_{R_{123}}$ β-continuous if $f^{-1}(H)$ is a nano $\tau_{R_{123}}\beta$-open set in U for every nano $\sigma_{R'_{123}}$open set H in V.

Definition 5.2 Let $\left(U, \tau_{R_{123}}(X)\right)$ and $\left(V, \sigma_{R'_{123}}(Y)\right)$ be a nano tritopological spaces. A function $f: \left(U, \tau_{R_{123}}(X)\right) \to \left(V, \sigma_{R'_{123}}(Y)\right)$ is said to be a $N\tau_{R_{123}} - \delta\beta$-continuous function if $f^{-1}(B)$ is a $N\tau_{R_{123}} - \delta\beta$-open set in U for every nano $N\sigma_{R'_{123}}$open set B in V.

The following proposition introduces the relationship between $N\tau_{R_{123}} - \beta$-continuous and $N\tau_{R_{123}} - \delta\beta$-continuous functions.

Proposition 5.1 Every $N\tau_{R_{123}} - \beta$-continuous is $N\tau_{R_{123}} - \delta\beta$-continuous.

Proof. Obvious.

The relationships between nano $\tau_{R_{123}}\delta\beta$-continuous function and the other different types of nano near continuous functions is shown in the following diagram,

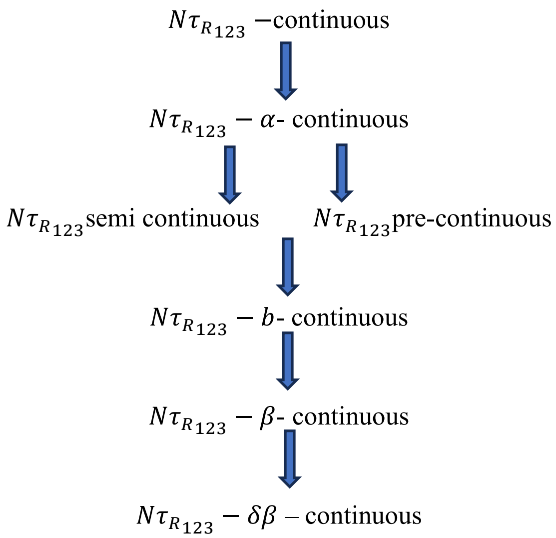

Figure 2: The relationships between the other different types of nano near continuous functions.

From **Proposition 5.1** and **Figure 2** we can notice that $N\tau_{R_{123}} - \delta\beta$- continuous function is a generalization of the other types of $N\tau_{R_{123}}$ near continues functions.

The converse of **Proposition 5.1** is not necessarily true as shown in the following example.

Example 5.1 Let $U = \{1,2,3,4\}$, $X = \{3,4\}$ with $U/R_1 = \{\{1,3\},\{2\},\{4\}\}$ then $\tau_{R_1}(X) = \{U, \emptyset, , \{4\}, \{1,3\}, \{1,3,4\}\}$, $U/R_2 = \{\{1\},\{3\},\{2,4\}\}$ then $\tau_{R_2}(X) = \{U, \emptyset, \{3\}, \{2,4\}, \{2,3,4\}\}$. $U/R_3 = \{\{1,3\},\{2,4\}\}$ then $\tau_{R_3}(X) = \{U, \emptyset\}$.

Hence, $N\tau_{R_{123}}(X) = \tau_{R_1}(X) \cup \tau_{R_2}(X) \cup \tau_{R_3}(X) = \{U, \emptyset, \{3\}, \{4\}, \{1,3\}, \{2,4\}, \{1,3,4\}, \{2,3,4\}\}$.

Let $V = \{a,b,c,d\}$, $Y = \{a,d\}$ with $V/R'_1 = \{\{a\},\{b,c\},\{d\}\}$ then, $\sigma_{R'_1}(Y) = \{V, \emptyset, \{a,d\}\}$, $V/R'_2 = \{\{a\},\{b\},\{c,d\}\}$ then, $\sigma_{R'_2}(Y) = \{V, \emptyset, \{a\}, \{c,d\}, \{a,c,d\}\}$. $V/R'_3 = \{\{b\},\{a,c,d\}\}$ then, $\sigma_{R'_3}(Y) = \{V, \emptyset, \{a,c,d\}\}$.

Hence, $N\sigma_{R'_{123}}(Y) = \sigma_{R'_1}(Y) \cup \sigma_{R'_2}(Y) \cup \sigma_{R'_3}(Y) = \{V, \emptyset, \{a\}, \{a,d\}, \{c,d\}, \{a,c,d\}\}$.

Define $f: U \to V$ as $f(1) = a, f(2) = d, f(3) = c, f(4) = b$. Then we have

a) f is $N\tau_{R_{123}} - \delta\beta$ continuous mapping.

b) f is not $N\tau_{R_{123}} - \beta$ continuous since $f^{-1}(\{a\}) = \{1\}$ which is not $N\tau_{R_{123}} - \delta\beta$ open set in U.

c) f is not $N\tau_{R_{123}} - \alpha$ continuous since $f^{-1}(\{a,d\}) = \{1,2\}$ which is not $N\tau_{R_{123}} - \alpha$ open set in U.

d) f is not $N\tau_{R_{123}} - b$ continuous since $f^{-1}(\{c,d\}) = \{2,3\}$ which is not $N\tau_{R_{123}} - b$ open set in U.

e) f is not $N\tau_{R_{123}}$ pre-continuous since $f^{-1}(\{a,c,d\}) = \{1,2,3\}$ which is not $N\tau_{R_{123}}$ pre-open set in U.

Theorem 5.1 Let $\left(U, \tau_{R_{123}}(X)\right)$ and $\left(V, \sigma_{R'_{123}}(Y)\right)$ be a nano tritopological spaces and let $f: \left(U, \tau_{R_{123}}(X)\right) \to \left(V, \sigma_{R'_{123}}(Y)\right)$ be a mapping. Then, the following statements are equivalent:

a) f is a $N\tau_{R_{123}} - \delta\beta$-continuous mapping.

b) The image of every $N\sigma_{R'_{123}}$closed set G in V is a $N\tau_{R_{123}} - \delta\beta$-closed set in U.

c) $f\left(N\tau_{R_{123}} - cl_{\delta\beta}(A)\right) \subseteq N\sigma_{R'_{123}} - cl\left(f(A)\right)$, $\forall A \subseteq U$.

d) $N\tau_{R_{123}} - cl_{\delta\beta}\left(f^{-1}(F)\right) \subseteq f^{-1}\left(N\sigma_{R'_{123}} - cl(F)\right), \forall F \subseteq V$.

e) $f^{-1}(N\sigma_{R'_{123}} - int(F)) *\subseteq N\tau_{R_{123}} - int_{\delta\beta}(f^{-1}(F)), \forall F \subseteq V$.

Proof.

1) $a) \Rightarrow b)$ Let f be a $N\tau_{R_{123}} - \delta\beta$-continues mapping and let G be a $N\sigma_{R'_{123}}$ closed set in V. then $V - G$ is $N\sigma_{R'_{123}}$open set in V. Since, f is a $N\tau_{R_{123}} - \delta\beta$-continues mapping. Then, $f^{-1}(V - G)$ is a $N\tau_{R_{123}} - \delta\beta$-open set in U. Since $f^{-1}(V - G) = U - f^{-1}(V)$, therefore $f^{-1}(V)$ is a $N\tau_{R_{123}} - \delta\beta$-closed set in U.

2) $b) \Rightarrow a)$ Let A be a $N\sigma_{R'_{123}}$open set in V. Then $f^{-1}(V - A)$ is a a $N\tau_{R_{123}} - \delta\beta$-closed set in U. Then $f^{-1}(A)$ is a $N\tau_{R_{123}} - \delta\beta$-open set in U. Therefore, f is a $N\tau_{R_{123}} - \delta\beta$-continuous mapping.

3) $a) \Rightarrow c)$ Let f be a $N\tau_{R_{123}} - \delta\beta$-continues mapping and let $A \subseteq U$. Since, f is a $N\tau_{R_{123}} - \delta\beta$-continuous, nano $\sigma_{R'_{123}} cl(f(A))$ is $N\sigma_{R'_{123}}$closed in V, $f^{-1}\left(N\sigma_{R'_{123}} - cl(F)\right)$ is a $N\tau_{R_{123}} - \delta\beta$-closed set in U. Since, $f(A) \subseteq N\sigma_{R'_{123}} - cl(f(A))$, $f^{-1}(f(A)) \subseteq f^{-1}(N\sigma_{R'_{123}} - cl(f(A))$, then $N\tau_{R_{123}} - cl_{\delta\beta}(A) \subseteq N\tau_{R_{123}} - cl_{\delta\beta}\left(f^{-1}(N\sigma_{R'_{123}} - cl(f(A))\right) = f^{-1}\left(N\sigma_{R'_{123}} - cl(f(A))\right)$. Thus, $N\tau_{R_{123}} - cl_{\delta\beta}(A) \subseteq f^{-1}(N\sigma_{R'_{123}} - cl(f(A))$. Therefore, $\left(N\tau_{R_{123}} - cl_{\delta\beta}(A)\right) \subseteq N\sigma_{R'_{123}} - cl(f(A)), \forall A \subseteq U$.

4) $c) \Rightarrow a)$ Let $f\left(N\tau_{R_{123}} - cl_{\delta\beta}(A)\right) \subseteq N\sigma_{R'_{123}} - cl(f(A)), \forall A \subseteq U$ and let F be a $N\sigma_{R'_{123}}$closed set in V. Then $f^{-1}(F) \subseteq U$. Thus, $f\left(N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(F))\right) \subseteq N\sigma_{R'_{123}} - cl\left(f(f^{-1}(F))\right) \subseteq N\sigma_{R'_{123}} - cl(F) = F$ that is $N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(F)) \subseteq f^{-1}(f(N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(F)))) \subseteq f^{-1}(F)$. Thus, $N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(F)) \subseteq f^{-1}(F)$, but $f^{-1}(F) \subseteq N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(F))$. Hence, $N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(F)) = f^{-1}(F)$. Therefore $f^{-1}(F)$ is a $N\tau_{R_{123}} - \delta\beta$-closed set in U. Therefore, f is a $N\tau_{R_{123}} - \delta\beta$-continues mapping.

5) $a) \Rightarrow d)$ Let f be a $N\tau_{R_{123}} - \delta\beta$-continuous mapping and $F \subseteq V$. Since $F \subseteq N\sigma_{R'_{123}} cl(F)$, then $f^{-1}(F) \subseteq f^{-1}\left(\text{nano } \sigma_{R'_{123}} cl(F)\right)$ and hence $N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(F)) \subseteq N\tau_{R_{123}} - cl_{\delta\beta}\left(f^{-1}\left(N\sigma_{R'_{123}} - cl(F)\right)\right) = f^{-1}\left(N\sigma_{R'_{123}} - cl(F)\right)$ as $N\sigma_{R'_{123}} - cl(F)$ is a $N\sigma_{R'_{123}}$closed set in V and f is a $N\tau_{R_{123}} - \delta\beta$-continues. Hence, $N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(F)) \subseteq f^{-1}\left(N\sigma_{R'_{123}} - cl(F)\right)$, $\forall F \subseteq V$.

6) $d) \Rightarrow a)$ Let $N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(F)) \subseteq f^{-1}\left(N\sigma_{R'_{123}} - cl(F)\right), \forall F \subseteq V$ and let G is $N\sigma_{R'_{123}}$ –closed set in V. Then, $N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(G)) \subseteq f^{-1}\left(N\sigma_{R'_{123}} - cl(G)\right) = f^{-1}(G)$. Hence, $N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(G)) \subseteq f^{-1}(G)$, but $f^{-1}(G) \subseteq N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(G))$. Hence, $f^{-1}(G) = N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(G))$. Therefore, f is a $N\tau_{R_{123}} - \delta\beta$-continues.

7) $a) \Rightarrow e)$ Let f be a $N\tau_{R_{123}} - \delta\beta$-continuous mapping and let $F \subseteq V$. Since, $N\sigma_{R'_{123}} - int(F)$ is a $N\sigma_{R'_{123}}$open set in V, then $f^{-1}(N\sigma_{R'_{123}} - int(F))$ is a $N\tau_{R_{123}} - \delta\beta$-open set in U. Therefore, $N\tau_{R_{123}} - int_{\delta\beta}\left(f^{-1}\left(N\sigma_{R'_{123}} - int(F)\right)\right) = f^{-1}(N\sigma_{R'_{123}} - int(F))$. Also, $N\sigma_{R'_{123}} - int(F) \subseteq F$ implies that $f^{-1}\left(N\sigma_{R'_{123}} - int(F)\right) \subseteq f^{-1}(F)$. Therefore, $N\tau_{R_{123}} - int_{\delta\beta}(f^{-1}(N\sigma_{R'_{123}} - int(F))) \subseteq N\tau_{R_{123}} - int_{\delta\beta}(f^{-1}(F))$. That is $f^{-1}(N\sigma_{R'_{123}} - int(F)) \subseteq N\tau_{R_{123}} - int_{\delta\beta}(f^{-1}(F))$, $\forall F \subseteq V$.

8) $e) \Rightarrow a)$ Let $f^{-1}(\text{nano } \sigma_{R'_{123}} int(F)) \subseteq \text{nano } \tau_{R_{123}} int_{\delta\beta}(f^{-1}(F)), \forall F \subseteq V$ and let G is nano$\sigma_{R'_{123}}$open set in V, then $N\sigma_{R'_{123}} - int(G) = G$. By assumption, $f^{-1}(N\sigma_{R'_{123}} - int(G)) \subseteq N\tau_{R_{123}} - int_{\delta\beta}(f^{-1}(G))$. Thus $f^{-1}(G) \subseteq N\tau_{R_{123}} - int_{\delta\beta}(f^{-1}(G))$. But $N\tau_{R_{123}} - int_{\delta\beta}(f^{-1}(G)) \subseteq f^{-1}(G)$. Hence, $f^{-1}(G) = N\tau_{R_{123}} - int_{\delta\beta}(f^{-1}(G))$. Therefore, f is a $N\tau_{R_{123}} - \delta\beta$-continues.

Remark 5.1 In **Theorem 5.1** the equality of parts c), d) and e) does not hold in general as shown in the following example.

Example 5.2 From **Example 5.1**,

in part c).

If $A = \{1,2\}$, then $f\left(N\tau_{R_{123}} - cl_{\delta\beta}(A)\right) = f(\{1,2\}) = \{a, d\}, N\sigma_{R'_{123}} - cl(f(A)) = N\sigma_{R'_{123}} - cl(\{a, d\}) = V$. Since $\{a, d\} \neq V$ then $f\left(N\tau_{R_{123}} - cl_{\delta\beta}(A)\right) \neq N\sigma_{R'_{123}} - cl(f(A))$.

In part d).

If $F = \{a\}$, then $N\tau_{R_{123}} - cl_{\delta\beta}(f^{-1}(\{a\})) = N\tau_{R_{123}} - cl_{\delta\beta}(\{1\}) = \{1\}, f^{-1}\left(N\sigma_{R'_{123}} - cl(\{a\})\right)$

$= f^{-1}(\{a,b\}) = \{1,4\}$. Since $\{1\} \neq \{1,4\}$ then $N\tau_{R_{123}} - cl_{\delta\beta}\left(f^{-1}(F)\right) \neq f^{-1}(N\sigma_{R'_{123}} - cl(F))$.

In part e).

If $F = \{a,b,c\}$, then $f^{-1}\left(N\sigma_{R'_{123}} - int(F)\right) = f^{-1}\left(N\sigma_{R'_{123}} - int(\{a,b,c\})\right) = f^{-1}(\{a\}) = \{1\}$, $N\tau_{R_{123}} - int_{\delta\beta}\left(f^{-1}(F)\right) = N\tau_{R_{123}} - int_{\delta\beta}\left(f^{-1}(\{a,b,c\})\right) = N\tau_{R_{123}} - int_{\delta\beta}(\{1,3,4\}) = \{1,3,4\}$. Since $\{1\} \neq \{1,3,4\}$ then $f^{-1}\left(N\sigma_{R'_{123}} - int(F)\right) \neq N\tau_{R_{123}} - int_{\delta\beta}(f^{-1}(F))$.

6. Conclusion

This paper introduced a study of new concept in nano tritopological spaces, which was $N\tau_{R_{123}} - \delta\beta$-open sets. The class of $N\tau_{R_{123}} - \delta\beta$-open sets was stronger than any type of the previous classes of $N\tau_{R_{123}}$- near open sets such as, $N\tau_{R_{123}}$-regular open set, $N\tau_{R_{123}}$-semi open set, $N\tau_{R_{123}}$-preopen set, $N\tau_{R_{123}} - \alpha$-open set and $\tau_{R_{123}} - \beta$-open set, etc. accordingly, $N\tau_{R_{123}} - \delta\beta$-open sets were generalized the usual notions of $N\tau_{R_{123}}-$ near open sets. Several topological characterizations and properties of the current new sort of sets were studied. Additionally, various forms of $N\tau_{R_{123}} - \delta\beta$-open sets corresponding to different cases of approximations were investigated. Moreover, the concepts of nano near continues functions were extended to $N\tau_{R_{123}} - \delta\beta$-continues functions. It was showed that every $N\tau_{R_{123}} - \beta$-continues function was $N\tau_{R_{123}} - \delta\beta$-continue functions. Therefore, the $N\tau_{R_{123}} - \delta\beta$-continues functions were generalization of the other types of $N\tau_{R_{123}}$ −near continues functions.

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